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THE AERODYNAMIC HEATING OF A
COMPOSITE FLAT PLATE

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Ballistics Research Report 113

THE AERODYNAMIC HEATING OF A COMPOSITE FLAT PLATE*

Prepared by:
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ABSTRACT: The purpose of this study was to find solutions to the heat-conduction equation in both one and two dimensions when applied to a composite flat plate. The two-dimensional model is that of two slabs with different thermal properties, in good thermal contact with each other while being exposed to the same boundary layer. The solution takes the form of Fourier series which has been found to be orthogonal with respect to a "weighting" function identical to the "weighting" function found by Mayer in the one-dimensional analysis of a composite slab. The Van Driest turbulent boundary-layer theory has been applied to the one-dimensional problem to determine the constant film coefficient boundary condition. It has been found that for those values of the Fourier modulus less than 0.1, a maximum error in surface temperature ratio of 8 percent may be expected while the temperature distribution in general is unaffected for the case considered.

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U. S. NAVAL ORDNANCE LABORATORY
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LIST OF SYMBOLS

a	width of region one
b	width of region two
c	specific heat of material
c_{f_∞}	coefficient of friction
h	film coefficient of heat transfer
ℓ	thickness of flat plate
q	heat flux per unit area per unit time
t	time
$\bar{t}$	Fourier modulus, $\kappa t/\ell^2$
U	free-stream velocity
X	coordinate parallel to free stream
y	coordinate perpendicular to free stream
C_p	specific heat at constant pressure of gas
K	coefficient of thermal conductivity of material
L	Biot modulus, $h\ell/K$
M	Mach number, ratio of velocity to the speed of sound
Q_0	constant heat flux per unit area per unit time
R	Reynolds number, $\rho_\infty U_\infty X/\mu_\infty$
T	temperature
T_{aw}	adiabatic wall temperature
α	constant defined by equation (27b)
β	constant defined by equation (27a)
γ	constant defined by equation (27a). Also, ratio of specific heats for an ideal gas

δ	constant defined by equation (27b)
η	recovery factor
κ	coefficient of thermal diffusivity of material
λ	constant defined by equation (25a)
μ	coefficient of viscosity of gas
ρ	density of gas; density of material
τ	shearing force per unit area
ϕ	orthogonal function defined by equation (A-2)
ψ	temperature function defined by equation (A-1)

SUBSCRIPTS

m	index referring to roots of equation (31a)
n	index referring to roots of equation (31b)
p	index
q	index
w	refers to wall condition, $y = 0$
1	refers to region one
2	refers to region two
∞	refers to free stream properties

INTRODUCTION

In recent years attention has been focused on the problems associated with the extremely high temperatures encountered by a vehicle during re-entry. A working knowledge of the nature of the heat-convection problem is required by the engineer so that the vehicle may be designed to function properly in such a high-temperature environment. As a result, the rate of heat transfer to the vehicle becomes an extremely important design parameter. However, since the nature of all aerodynamic heating problems is not yet fully understood, most engineers must resort to experimental data for the prediction of the various heating parameters.

Experimental data may be obtained by various methods, such as, wind tunnel or shocktube tests. In both types of testing, an attempt is made to simulate both the geometry and environment the full scale vehicle might be expected to experience. The heat-transfer parameters are then calculated from some knowledge of the temperature history of the model.

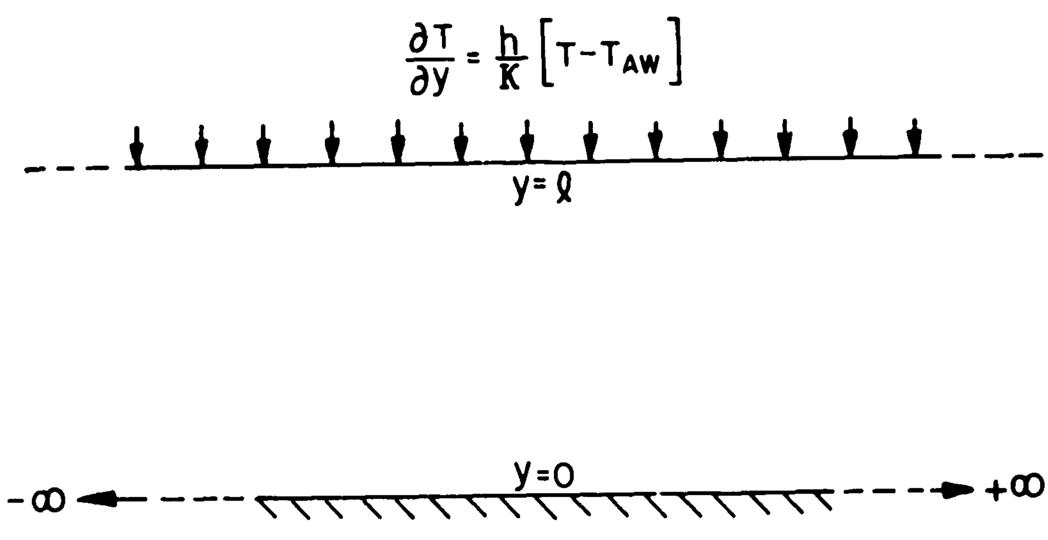
One of the more popular models (fig. 1) used to obtain experimental data is the smooth flat plate placed in the direction of a high-temperature uniform flow in which the pressure is constant. The temperature distribution in the plate is then given by the theory of heat conduction, where the boundary conditions at the surface exposed to the flow are given by the various boundary-layer theories. Since this provides one of the simplest of mathematical models it is possible to study the nature of the phenomenon more accurately.

Normally, the heat-conduction problem may be considered to be one dimensional with boundary conditions of varying degrees of complexity. For relatively low rates of heat transfer or short periods of heating (ref. (1)), the flat plate may be considered to be a semi-infinite solid. This arises from the fact that the total heat flux is not sufficient to penetrate the plate during the period of the test. As the thickness of the plate is reduced, the mathematical model must be changed to that of a finite slab with various boundary conditions on the rear surfaces. As a practical application, the temperature distribution in a calorimeter gage (ref. (2)) is obtained when this boundary condition is that of zero heat flux. When the thickness of the plate is reduced still further, the temperature may be considered constant. Utilizing this boundary condition, the response of a thin film-resistance thermometer may be obtained. Carslaw and Jaeger (ref. (3)) have described most of these conditions and their corresponding solutions by various methods in their text.

The shocktube is often used to produce extremely high rates of heat transfer because of its ability to generate extremely high enthalpies and temperatures. Heat-transfer rates may be obtained, as usual, by imbedding a calorimeter gage in the flat plate exposed to the flow. However, due to the extremely high pressures associated with this type of test, severe problems are encountered in bonding and sealing the gage into position. As a result, the gage must be made much thicker than normal and the resulting area in thermal contact with the bonding material becomes quite large. The heat flow in the gage may become two dimensional depending on the thermal properties of the bonding material. In order to determine the temperature distribution in the gage the flat plate is assumed to be formed by two dissimilar materials in good thermal contact as shown in figure 1. Since both materials are exposed to the same boundary layer and to each other, a solution to the heat-conduction equation in two dimensions must be developed. With this solution, the effect of the aerodynamic heating of a composite flat plate may be calculated.

ONE-DIMENSIONAL ANALYSIS

It has been mentioned in the preceding section that the temperature distribution of a calorimeter gage may be calculated from the theory of heat conduction in a slab of finite thickness. If the flow of heat may be considered one dimensional, the mathematical model used for this calculation is shown in the following sketch.



The differential equation governing the flow of heat is given by:

$$\chi \frac{\partial^2 T}{\partial y^2} - \frac{\partial T}{\partial t} = 0 \qquad o < y < \ell \qquad (1)$$

Where T = T (y,t) is the temperature above the initial temperature of the gage (i.e. T = 0 when t = o). At times greater than zero, the temperature at the surface is governed by the equation:

$$K \left.\frac{\partial T}{\partial y}\right|_{y=\ell} + h\left[T_{AW} - T(\ell, t)\right] = 0 \qquad t > 0 \qquad (2)$$

The flux is, therefore, proportional to the difference between the surface temperature and some temperature determined by the surrounding medium. The proportionality constant "h" is the film coefficient of heat transfer which is assumed constant for this analysis.

At the back surface the heat flux is assumed to be zero, therefore:

$$\left.\frac{\partial T}{\partial y}\right|_{y=o} = 0 \qquad t > o \qquad (3)$$

Let us assume a solution of the form:

$$T(y,t) = g(y)e^{-\lambda^2 t} + C \qquad (4)$$

Upon substitution into equation (1) we obtain a differential equation in g(y) which has as its solution:

$$g(y) = A \cos \frac{\lambda y}{\sqrt{\chi}} + B \sin \frac{\lambda y}{\sqrt{\chi}} \qquad (5)$$

By combining equations (4) and (5) and applying the boundary condition at y = 0, we see that:

$$B = 0 \qquad (6)$$

Upon applying the boundary condition at y = 1 we have, after some rearrangement:

$$\beta \tan \beta - L = 0 \qquad (7)$$

and

$$C = T_{AW} \qquad (8)$$

where $\beta = \frac{\lambda \ell}{\sqrt{\chi}}$ and $L = \frac{h \ell}{K}$, the Biot modulus

Then

$$T(y,t) = T_{AW} + \sum_{N=1}^{\infty} A_N \cos \frac{\beta_N y}{\ell} e^{-\beta_N^2 \tau} \qquad (9)$$

where the β_n's are the positive roots of equation (7), and $\tau = \chi t/\ell^2$, the Fourier modulus, a non-dimensional time. The coefficients A_n must be determined from a Fourier analysis of the series at time zero. Therefore:

$$\sum_{N=1}^{\infty} A_N \cos \frac{\beta_N y}{\ell} = - T_{AW} \qquad t=0 \qquad (10)$$

By Fourier analysis:

$$A_N = - T_{AW} \frac{\int_0^{\ell} \cos \frac{\beta_N y}{\ell} dy}{\int_0^{\ell} \cos^2 \frac{\beta_N y}{\ell} dy} \qquad (11)$$

or finally:

$$A_N = -2 T_{AW} \frac{\sin \beta_m}{\beta_N + \sin \beta_N \cos \beta_N} \qquad (12)$$

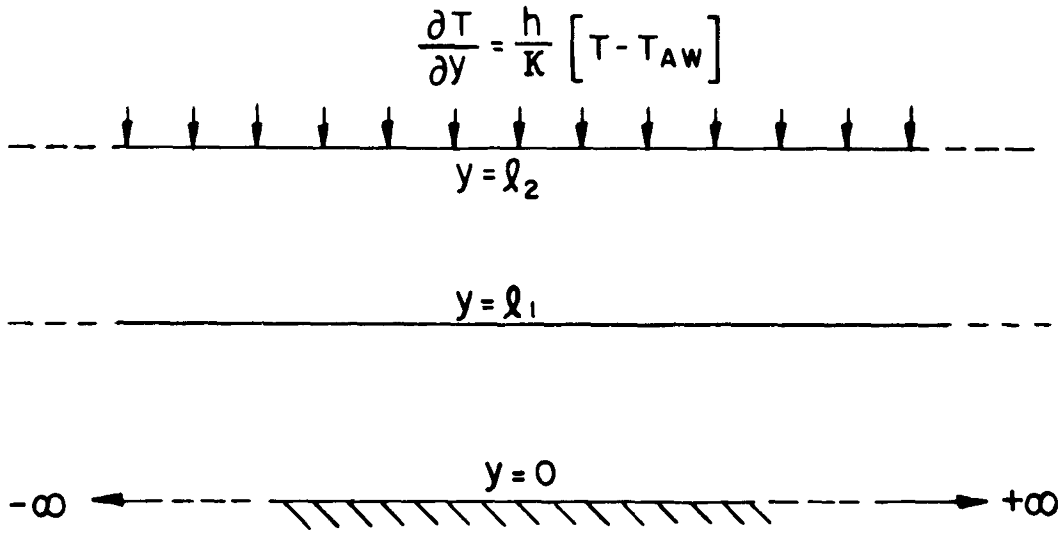
This solution is a special case of the solution obtained by Carslaw and Jaeger to the problem of the finite slab with arbitrary-boundary conditions.

A more exact analysis of this problem is sometimes achieved by taking into account the loss of heat at the back surface. This is done by assuming a second dissimilar material to be in good thermal contact with the back of the gage. At the interface of the two materials both the temperature and heat flux must be identical, therefore:

$$T_1 = T_2 \qquad (13)$$

and
$$K_1 \frac{\partial T_1}{\partial y} = K_2 \frac{\partial T_2}{\partial y} \tag{14}$$

The second material may then be thought of as a semi-infinite solid or another finite slab as shown in the following sketch.



This problem has been treated by many authors since it is an excellent mathematical model for a variety of engineering problems. In most treatments of this problem the film coefficient is again considered to be constant. Mayer (ref. (4)) has obtained a solution by the methods of Fourier analysis which is very useful for large values of time. Wasserman (ref. (5)) and Campbell (ref. (6)) have obtained solutions for small values of time for this problem.

If the heating rate or testing time is extremely short, the surface temperature will not change appreciably and the boundary condition given by equation (2) may be approximated by:

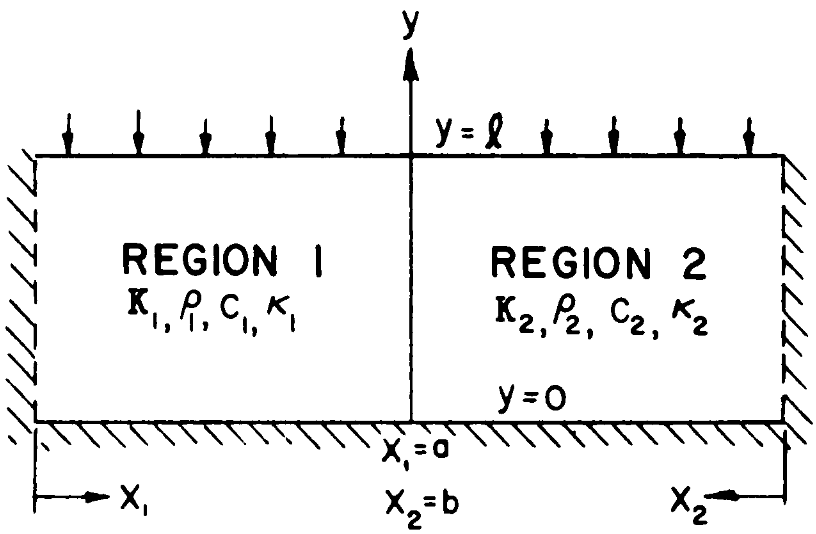
$$-K \left.\frac{\partial T}{\partial y}\right|_{y=\ell} = Q_o \tag{15}$$

where Q_o is the heat flux at time zero, and is considered to be constant. Rose (ref. (2)) has used this boundary condition and solved the problem by LaPlace transform methods to obtain short-time solutions which normally do not have the convergence problems associated with the Fourier analysis.

TWO-DIMENSIONAL ANALYSIS

The one-dimensional solutions previously discussed are excellent treatments of the heat-conduction problem when the y-dimension is much smaller than the x-dimension. However, when these dimensions are of the same order of magnitude, care must be taken to consider any possible end effects.

Therefore, consider the composite flat-plate system shown in figure 1. If we restrict our problem to that of finding the temperature distribution for small values of time, we may consider at the back surface, the heat flux in the y-direction to be everywhere zero and at the front surface the film coefficient to be constant. Then by considering the symmetry of the system,the heat flux at the center of each element is zero, and we may reduce our problem to that shown in the following sketch.



The differential equations governing the flow of heat are given by:

$$\kappa_1 \frac{\partial^2 T_1}{\partial x_1^2} + \kappa_1 \frac{\partial^2 T_1}{\partial y^2} - \frac{\partial T_1}{\partial t} = 0 \qquad (16a)$$

and

$$\kappa_2 \frac{\partial^2 T_2}{\partial x_2^2} + \kappa_2 \frac{\partial^2 T_2}{\partial y^2} - \frac{\partial T_2}{\partial t} = 0 \qquad (16b)$$

where $T = T(x,y,t)$ is the temperature above ambient and is initially zero throughout.* The boundary conditions for this problem are as follows:

In region one:

$$\left.\frac{\partial T_1}{\partial x_1}\right|_{(o,y,t)} = 0 \tag{17}$$

$$\left.\frac{\partial T_1}{\partial y}\right|_{(x,o,t)} = 0 \tag{18}$$

$$K_1 \left.\frac{\partial T_1}{\partial y}\right|_{(x_1,\ell,t)} + h\left[T_1(x_1,\ell,t) - T_{Aw}\right] = 0 \tag{19}$$

In region two:

$$\left.\frac{\partial T_2}{\partial x_2}\right|_{(o,y,t)} = 0 \tag{20}$$

$$\left.\frac{\partial T_2}{\partial y}\right|_{(x_2,o,t)} = 0 \tag{21}$$

$$K_2 \left.\frac{\partial T_2}{\partial y}\right|_{(x_2,\ell,t)} + h\left[T_2(x_2,\ell,t) - T_{Aw}\right] = 0 \tag{22}$$

And at the interface:

$$T_1(a,y,t) = T_2(b,y,t) \tag{23}$$

$$K_1 \left.\frac{\partial T_1}{\partial x_1}\right|_{(a,y,t)} = -K_2 \left.\frac{\partial T_2}{\partial x_2}\right|_{(b,y,t)} \tag{24}$$

Seeking solutions by separation of variables let us assume solutions of the form:

$$T_1(x_1,y,t) = f_1(x)\, g_1(y)\, e^{-\lambda^2 t} + C_1 \tag{25a}$$

---

*The coordinate system has been selected as shown in figure 8, such that at $x_n = o$, the heat flux is zero.

and $$T_2(x_2, y, t) = f_2(x)\, g_2(y)\, e^{-\lambda^2 t} + C_2 \tag{25b}$$

Substitution of these functions into equations (16) yield:

$$\frac{f_1''}{f_1} + \frac{g_1''}{g_1} + \frac{\lambda^2}{\varkappa_1} = 0 \tag{26a}$$

and $$\frac{f_2''}{f_2} + \frac{g_2''}{g_2} + \frac{\lambda^2}{\varkappa_2} = 0 \tag{26b}$$

Since f(x) and g(y) are functions of different independent variables, each part of equations (26) must equal a constant; therefore:

$$\frac{f_1''}{f_1} = -\left(\frac{\gamma}{a}\right)^2 \quad \text{say, and} \quad \frac{g_1''}{g_1} = -\left(\frac{\beta}{\ell}\right)^2 \tag{27a}$$

and similarly $$\frac{f_2''}{f_2} = -\left(\frac{\delta}{b}\right)^2 \text{ say, and } \quad \frac{g_2''}{g_2} = -\left(\frac{\alpha}{\ell}\right)^2 \tag{27b}$$

then we obtain:

$$f_1(x) = A_1 \cos\gamma\frac{x}{a} + B_1 \sin\gamma\frac{x}{a} \tag{28}$$

and $$g_1(y) = E_1 \cos\beta\frac{y}{\ell} + D_1 \cos\beta\frac{y}{\ell} \tag{29}$$

and similar solutions for $f_2(x)$ and $g_2(y)$. It is apparent from equations (17) and (18) that $B_1 = D_1 = 0$. Likewise, it may be seen that $B_2 = D_2 = 0$. From equations (19) and (22), we may conclude that $C_1 = C_2 =$ Taw, so that the time dependent term can be canceled yielding:

$$K_1 \left.\frac{dg_1}{dy}\right|_{y=\ell} + h\, g_1(\ell) = 0 \tag{19'}$$

and $$K_2 \left.\frac{dg_2}{dy}\right|_{y=\ell} + h\, g_2(\ell) = 0 \tag{22'}$$

Substituting equations (29) into (19') and (22') we obtain:

$$E_1\left[\beta \sin\beta - L_1 \cos\beta\right] = 0 \tag{30a}$$

and $$E_2\left[\alpha \sin \alpha - L_2 \cos\alpha\right] = 0 \tag{30b}$$

Where $L_n = \frac{h\ell}{K_n}$ is again the Biot modulus, the constants E_1 and E_2 cannot equal zero lest the solution become one-dimensional, therefore:

$$\beta \tan \beta - L_1 = 0 \tag{31a}$$

and $$\alpha \tan \alpha - L_2 = 0 \tag{31b}$$

Let us now rewrite equations (25) with the help of (28) and (29):

$$T_1(x,y,t) = A \cos \gamma \frac{x}{a} \cos\beta \frac{y}{\ell} e^{-\lambda^2 t} + T_{AW} \tag{32a}$$

$$T_2(x,y,t) = B \cos \delta \frac{x}{b} \cos \alpha \frac{y}{\ell} e^{-\lambda^2 t} + T_{AW} \tag{32b}$$

From equations (23) and (24) we have:

$$A \cos \gamma \;\; \cos\beta \frac{y}{\ell} = B \cos \delta \;\; \cos \alpha \frac{y}{\ell} \tag{23'}$$

$$-A K_1 \frac{\gamma}{a} \sin\gamma \cos\beta \frac{y}{\ell} = B K_2 \frac{\delta}{b} \sin \delta \; \cos\alpha \frac{y}{\ell} \tag{24'}$$

Dividing (24') by (23') we obtain:

$$K_1 b \gamma \tan\gamma = -K_2 a \delta \tan \delta \tag{33}$$

From equations (26) and (27) we may write:

$$\varkappa_1 \left[\left(\frac{\gamma}{a}\right)^2 + \left(\frac{\beta}{\ell}\right)^2\right] = \varkappa_2 \left[\left(\frac{\delta}{b}\right)^2 + \left(\frac{\alpha}{\ell}\right)^2\right] \tag{34}$$

We may now write our solution in its final form:

$$T_1(x,y,t) = T_{AW} + \sum_{M=1}^{\infty}\sum_{N=1}^{\infty} A_{MN} \cos \gamma_{MN}\frac{x_1}{a}\cos\beta_M \frac{y}{\ell} e^{-\lambda^2_{MN}t} \tag{35a}$$

and
$$T_2(x,y,t) = T_{AW} + \sum_{M=1}^{\infty}\sum_{N=1}^{\infty} B_{MN} \cos \delta_{MN}\frac{x_2}{b}\cos\alpha_N \frac{y}{\ell} e^{-\lambda^2_{MN}t} \tag{35b}$$

where the α_N's and β_M's are the positive roots of equations (31), and the γ_{MN}'s and δ_{MN}'s are determined from the simultaneous solution of equations (33) and (34). The λ^2_{MN}'s may be determined from (26a) and (27a) as:

$$\lambda^2_{MN} = \frac{\kappa_1}{\ell^2}\left[\left(\frac{\ell}{a}\gamma_{MN}\right)^2 + \beta_M{}^2\right] \tag{36}$$

The coefficients of the series indicated in equations (35) must be determined from a Fourier analysis of the entire composite flat-plate system at time zero.

In order to expand the initial temperature into a Fourier series, some orthogonality relation must be determined. It has been shown in Appendix A that:

$$\int_0^\ell\int_0^a \rho_1 c_1\left[\cos\gamma_{MN}\frac{x_1}{a}\cos\beta_M\frac{y}{\ell}\right]\left[\cos\gamma_{pq}\frac{x_1}{a}\cos\beta_p\frac{y}{\ell}\right]dx\,dy \;+$$

$$\int_0^\ell\int_0^b \rho_2 c_2\left[\cos\delta_{MN}\frac{x_2}{b}\cos\alpha_N\frac{y}{\ell}\right]\left[\cos\delta_{pq}\frac{x_2}{b}\cos\alpha_q\frac{y}{\ell}\right]dx\,dy = 0 \tag{37}$$

Therefore, evaluating equation (35) at time zero, multiplying by the orthogonal function, and integrating over the entire composite-wall system we obtain:

$$\int_0^\ell\int_0^a A^2_{MN}\,\rho_1 c_1\left[\cos\gamma_{MN}\frac{x_1}{a}\cos\beta_M\frac{y}{\ell}\right]^2 dx\,dy + \int_0^\ell\int_0^b B^2_{MN}\,\rho_2 c_2\left[\cos\delta_{MN}\frac{x_2}{b}\cos\alpha_N\frac{y}{\ell}\right]^2 dx\,dy$$

$$= -T_{AW}\int_0^\ell\int_0^a A_{MN}\rho_1 c_1\left[\cos\gamma_{MN}\frac{x_1}{a}\cos\beta_M\frac{y}{\ell}\right]dx\,dy - T_{AW}\int_0^\ell\int_0^b B_{MN}\,\rho_2 c_2\left[\cos\delta_{MN}\frac{x_2}{b}\cos\alpha_N\frac{y}{\ell}\right]dx\,dy \tag{38}$$

From equation (23') we may write:

$$B_{MN} = \frac{\cos\gamma_{MN}\;\cos\beta_M\frac{y}{\ell}}{\cos\delta_{MN}\;\cos\alpha_N\frac{y}{\ell}}\,A_{MN} \tag{39}$$

Substituting equation (39) into (38) yields the following:

$$A_{mn}\left[\rho_1 c_1 C_{1mn} + \left(\frac{\cos\gamma_{mn}}{\cos\delta_{mn}}\right)^2 \rho_2 c_2 C_{2mn}\right] =$$

$$-T_{AW}\left[\rho_1 c_1 D_{1mn} + \left(\frac{\cos\gamma_{mn}}{\cos\delta_{mn}}\right)\rho_2 c_2 C_{2mn}\right. \tag{40}$$

where $$C_{1mn} \equiv \int_0^{\ell}\int_0^{a}\left[\cos\gamma_{mn}\frac{x}{a}\cos\beta_m\frac{y}{\ell}\right]^2 dx\,dy \tag{41}$$

$$C_{2mn} \equiv \int_0^{\ell}\int_0^{b}\left[\cos\delta_{mn}\frac{x}{b}\cos\beta_m\frac{y}{\ell}\right]^2 dx\,dy \tag{42}$$

$$D_{1mn} \equiv \int_0^{\ell}\int_0^{a}\left[\cos\gamma_{mn}\frac{x}{a}\cos\beta_n\frac{y}{\ell}\right] dx\,dy \tag{43}$$

$$D_{2mn} \equiv \int_0^{\ell}\int_0^{b}\left[\cos\delta_{mn}\frac{x}{b}\cos\beta_m\frac{y}{\ell}\right] dx\,dy \tag{44}$$

Now performing the integration we have:

$$C_{1mn} = \frac{a\ell}{4\gamma_{mn}\beta_m}\left\{\left[\beta_m + \sin\beta_m\cos\beta_m\right]\left[\gamma_{mn} + \sin\gamma_{mn}\cos\gamma_{mn}\right]\right\} \tag{41'}$$

$$C_{2mn} = \frac{b\ell}{4\delta_{mn}\beta_m}\left\{\left[\beta_m + \sin\beta_m\cos\beta_m\right]\left[\delta_{mn} + \sin\delta_{mn}\cos\delta_{mn}\right]\right\} \tag{42'}$$

$$D_{1mn} = \frac{a\ell}{\beta_m\gamma_{mn}}\sin\gamma_{mn}\,\sin\beta_m \tag{43'}$$

$$D_{2mn} = \frac{b\ell}{\beta_m\delta_{mn}}\sin\delta_{mn}\,\sin\beta_m \tag{44'}$$

Finally, the temperature is obtained by substituting equation (36) into (35) to obtain:

$$T_1(x,y,t) = T_{AW} + \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} A_{mn}\cos\gamma_{mn}\frac{x_1}{a}\cos\beta_m\frac{y}{\ell}\,e^{-\left[\left(\frac{\ell}{a}\gamma_{mn}\right)^2+\beta_m^2\right]\bar{t}_1} \tag{35'a}$$

where $\bar{t}_1 = \frac{\chi_1 t}{\ell^2}$, is again the Fourier modulus.

The coefficients $A_{m,n}$ are obtained from equation (40) after some rearrangement as:

$$A_{MN} = -T_{AW} \left[\frac{4 \sin \beta_m}{\beta_m + \sin \beta_m \cos \beta_m} \right] X$$

$$\frac{\sin \gamma_{mn} + \left(\frac{\rho_2 c_2 b \gamma_{mn}}{\rho_1 c_1 a \delta_{mn}}\right) \tan \delta_{mn} \cos \gamma_{mn}}{(\gamma_{mn} + \sin \gamma_{mn} \cos \gamma_{mn}) + \left(\frac{\rho_2 c_2 b \gamma_{mn}}{\rho_1 c_1 a \delta_{mn}}\right)\left(\frac{\cos \gamma_{mn}}{\cos \delta_{mn}}\right)^2 (\delta_{mn} + \sin \delta_{mn} \cos \delta_{mn})} \quad (45)$$

One possible method of determining the validity of this solution is to allow the material properties in region two to approach the material properties in region one. When this is done the composite flat plate behaves as a single slab and the above solution should reduce to the one-dimensional solution previously obtained.

It can be seen from equations (31) that if $L_1 = L_2$, then $\alpha_n = \beta_m$. Applying this condition to equation (34) we see that $b\gamma = a\delta$. However, since a and b may take on any values, $\gamma = \delta = 0$. Substituting these values into equations (35'a) and (45) we obtain:

$$T_1(x,y,t) = T_{AW} + \sum_{M=1}^{\infty} A_{mn} \cos \beta_m \frac{y}{\ell} e^{-\beta_m^2 \bar{t}_1} \quad (46)$$

and

$$A_{mn} = -T_{AW} \left[\frac{4 \sin \beta_m}{\beta_m + \sin \beta_m \cos \beta_m} \right] X \left[\frac{1}{2}\right] \quad (47)$$

which is in fact the one-dimensional solution previously obtained.

DETERMINATION OF EIGENVALUES

The temperature distribution in a composite flat plate is a function of x, y, and t and is obtained by evaluating the double Fourier series derived in the preceeding section. The indexing scheme is based on the real, positive roots of the transcendental equations (31). These equations have been studied in great detail by a number of authors. Carslaw and Jaeger (ref. (3)), for example, have tabulated the first 6 roots of these equations for a wide range of values of the Biot modulus, "L".

However, in order to evaluate equations (35'a), a third index, l, must be employed since the simultaneous solution of equations (33) and (34) does not yield unique values for γ_{mn} or δ_{mn}. For the purpose of illustration, these equations have been presented in figure 2, assuming the physical constants of

the composite system to be known. Using only the first root of (31a) and the first 6 roots of (31b), equation (34) has been evaluated and is presented as the family of curves represented by solid lines. Equation (33) is presented as the family of curves represented by the broken lines. The intersection of these two families represents the simultaneous solution of these equations. It is obvious that any number of values may be obtained for γ_{mj} and δ_{mj} which will satisfy these equations. Therefore, it is necessary, when evaluating equations (35a) and (45), to introduce a third indexing scheme in order to distinguish between the intersections of each of the solid curves shown in figure 2.

HEAT TRANSFER THROUGH THE BOUNDARY LAYER

As has been previously mentioned, the temperature distribution through a flat plate is dependent primarily on the film coefficient of heat transfer. In fact, it is common practice to deduce this coefficient by measurement of the temperature distribution; therefore, some information as to the character of this parameter should be reviewed.

Heat transfer to the flat plate is caused by the presence of a boundary layer which was first reported by Prandtl (ref. (7)) in 1904 and today has become the most important branch of the fluid dynamics of viscous flow (ref. (8)). Many researchers have considered this phenomenon; in fact, Eckert (ref. (9)) cites 439 separate contributions to the heating problem alone. We will restrict our consideration to the "turbulent" boundary layer since this produces the most severe-heating conditions during high-speed flight. The presence of this boundary layer produces a shearing force at the surface of the flat plate given by:

$$\tau_w \equiv \mu_w \left.\frac{\partial u}{\partial y}\right|_w \tag{48}$$

where μ_w is the coefficient of viscosity at the surface.

A coefficient of friction is then defined on the basis of this quantity and the free-stream conditions as:

$$c_{f_\infty} \equiv \frac{2\tau_w}{\rho_\infty U_\infty^2} \tag{49}$$

The heat flux at the surface is given by the theory of heat conduction as:

$$q_w = K_w \left.\frac{\partial T}{\partial y}\right|_w \tag{50}$$

Finally the film coefficient is defined in terms of the heat flux as:

$$h = \frac{q_w}{(T_{Aw} - T_w)} \tag{51}$$

where $T_{Aw} = T_\infty\left(1+\eta\frac{(\gamma-1)}{2}M_\infty^2\right)$ and η is a recovery factor. This is essentially the temperature the surface might be expected to attain when insulated.

It has been reported (ref. (10)) that the Van Driest turbulent boundary-layer theory (ref. (11)) for a flat plate predicts local wall friction coefficients which agree well with experimental data. Therefore, this theory will be used as an illustration of the dependence of the film coefficient on various aerodynamic parameters. Assuming a Prandtl number of unity, Van Driest has derived, along with others, that the heat transfer at the wall may be related to the shearing force as follows:

$$h = \frac{c_p \tau_w}{\mu_\infty} \tag{52}$$

Then based on Prandtl's mixing-length theory and knowledge of certain limiting conditions, Van Driest obtained his final equation for heat transfer through a turbulent-boundary layer in terms of the local coefficient of friction:

$$\frac{0.242}{A}\left(c_{f_\infty}\frac{T_w}{T_\infty}\right)^{-\frac{1}{2}}\left\{\sin^{-1}\frac{A - B/2A}{\left[(B/2A)^2+1\right]^{1/2}} + \sin^{-1}\frac{B/2A}{\left[(B/2A)^2+1\right]^{1/2}}\right\} = \tag{53}$$

$$0.41 + \log_{10} R_\infty c_{f_\infty} - 0.76 \log_{10} \frac{T_w}{T_\infty}$$

where $A^2 = \frac{\gamma-1}{2}M_\infty^2(T_\infty/T_w)$; and $B=\left(1+\frac{\gamma-1}{2}M_\infty^2\right)\frac{T_\infty}{T_w} - 1$
Therefore, the above equation allows the coefficient of friction and therefore the film coefficient of heat transfer to be computed for any arbitrary wall-temperature ratio, Reynolds number and free-stream Mach number.

It should be noted that the analyses in the preceding sections were based on the assumption that the film coefficient of heat transfer remains constant during the period of investigation. However, it has been shown that this coefficient is a function of not only the aerodynamic parameters of Mach number and Reynolds number but also the surface temperature of the flat plate. Therefore, great care must be taken in using this type of analysis.

In order to see what effect this restriction might have on the results of the preceding section, the boundary-layer theory must be incorporated with the heat-conduction analysis. Therefore, let us consider equations (9) and (12) and apply the results obtained by Van Driest in equation (53). Assuming a Mach number of unity equation (53) has been evaluated and is presented in figure 3 with Reynolds number as the parameter. Then by assuming a Reynolds number and free-stream temperature, the film coefficient has been evaluated (refs. (12) and (13)) and is presented in figure 4 for wall-temperature ratios less than unity. This is the case of the "highly cooled" boundary layer and gives rise to the most severe-heating rates. The film coefficient based on the initial wall-temperature ratio may then be obtained directly from figure 4 and the temperature distribution through the flat plate obtained by evaluating equations (9) and (12).

Assuming the plate to be initially at room temperature, the temperature distribution has been calculated for various values of the Fourier modulus and is presented in figure 5 as the family of curves represented by broken lines. It can be seen that significant changes in the wall temperature and therefore film coefficient occur even for relatively small values of the Fourier modulus. In an effort to see what effect these changes might have, the boundary layer was assumed to react instantaneously to these changes in wall temperature. It has been reported (ref. (14)) that this assumption will give satisfactory results. Equations (9) and (12) have been re-evaluated by allowing the film coefficient to change as the surface temperature of the plate increases. This scheme produces the temperature distribution shown in figure 5 as the family of curves represented by solid lines. Repeating this procedure does little to increase the accuracy of the calculation as can be seen in figure 6.

DISCUSSION OF RESULTS AND CONCLUSIONS

In the preceding section we have discussed the effect of a variable surface temperature on the temperature distribution in

the one-dimensional case. When considering the composite flat plate it is possible not only for the surface temperatures of the two materials to change at different rates due to the difference in thermal properties, but the character of the boundary layer itself may change. Regardless of the cause of the change in film coefficient of heat transfer, the effect in the two-dimensional case may be found in the same manner as in the one-dimensional case. It can be seen from equations (31) that a change in "h" has the same effect as a change in the thermal conductivity which alters the Biot modulus and thereby changes the roots of the transcendental equation. It is possible to apply the iterative scheme previously described to the two-dimensional solution to determine this effect; however, a simplified one-dimensional analysis based on the maximum Biot modulus is sufficient to determine the maximum error.

In conclusion it may be said that for the one-dimensional case considered, a maximum error in temperature ratio of less than 8 percent may be expected for Fourier moduli less than 0.1 and less than 6 percent for a modulus of 0.03, by assuming the film coefficient of heat transfer constant and equal to its initial value. This error is apparent only at the surface of the plate and the temperature distribution is essentially unchanged. The various parameters assumed in this report correspond to typical flow conditions attainable in a reflected shocktube and therefore represent extremely severe-heating conditions. Assuming the physical dimensions and properties of a typical calorimeter gage, the Fourier modulus of 0.03 would correspond to a time of approximately 300 microseconds which is well within the range of current measuring instruments. In wind-tunnel tests much lower free-stream temperatures are present and, therefore, it is conceivable that the constant film-coefficient condition may be used for even larger values of the Fourier modulus. In reality, the calculated temperatures will always be slightly greater than the actual temperatures and will therefore lead to a conservative design analysis.

The equations derived in this report for the temperature distribution in a flat plate in both one and two dimensions have employed a Fourier analysis to determine the coefficients of an infinite series. For small values of the Fourier modulus, these series tend to converge quite slowly, making hand calculations very impractical. However, the solutions presented here could be incorporated into a machine program for the reduction of experimental data or prediction of heat-transfer parameters.

REFERENCES

(1) Cresci, R. J., and Libby, P. A., "Some Heat Conduction Solutions Involved in Transient Heat-Transfer Measurements," WADC Technical Note No. 57-236, September 1957

(2) Rose, P. H., "Development of the Calorimeter Heat-Transfer Gage for Use in Shock Tubes," AVCO Research Report 17, AVCO Research Laboratory, 1958

(3) Carslaw, H. S., and Jaeger, J. C., Conduction of Heat in Solids, 2nd Ed., Oxford University Press, 1959

(4) Mayer, E., "Heat Flow in Composite Slabs," Journal of the American Rocket Society, Vol. 22, No. 3, May-June 1952

(5) Wasserman, B., "One-Dimensional Transient Heat Conduction into a Double-Layer Slab Subjected to a Linear Heat Input for a Small Time Interval," Journal of Aerospace Sciences, Vol. 24, pp. 924-925, December 1957

(6) Campbell, W. F., "One-Dimensional Transient Heat Flow in a Multilayer Slab," Journal of Aerospace Sciences, Vol. 25, pp. 340-341, May 1958

(7) Prandtl, L., "Uber Flussigketisbewegung bei sehr kleiner Reibung." Proceedings 3rd Intern. Math. Congr., Heidelberg 1904. Reprinted as NACA TM 452 (1928)

(8) Pai, S. I., Introduction to the Theory of Compressible Flow, D. Von Nostrand Company, Inc., Princeton, N. J. 1959, p. 328

(9) Eckert, E. R. G., "Survey of Boundary Layer Heat Transfer at High Velocities and High Temperatures," WADC Tech. Rept. 59-624, 1960

(10) Lee, D. B., and Faget, M. A., "Charts Adapted from Van Driest's Turbulent Flat-Plate Theory for Determining Values of Turbulent Aerodynamic Friction and Heat-Transfer Coefficients," NACA TN 3811, 1956

(11) Van Driest, E. R., "Turbulent Boundary Layer in Compressible Fluids," Journal of the Aeronautical Science, Vol. 18, No. 3, March 1951, pp 145-160, 216

(12) Hansen, C. F., "Approximations for the Thermodynamic and Transport Properties of High-Temperature Air," NASA TR-R-50, 1959

(13) Feldman, S., Hypersonic Gas Dynamic Charts for Equilibrium Air, AVCO Research Laboratory, 1957

(14) Eckert, E. R. G., "Survey on Heat Transfer at High Speeds," WADC Technical Report 54-70, 1954

APPENDIX A

AN ORTHOGONALITY RELATION

The decisive property of a set of functions which makes it possible to determine one by one the coefficients in the assumed expansion is that the integral of the product of any two distinct members of the set taken over the appropriate interval is zero. The set of functions is then said to be orthogonal (ref. (3)). It may be seen that the functions, $\psi_{mn}(x,y)$ described by equations (35), are not themselves orthogonal.

$$\psi_{mn}(x,y) = \begin{cases} f_{1_{mn}}(x)\, g_{1_m}(y) = A_{mn} \cos \gamma_{mn} \frac{x}{a} \cos \beta_m \frac{y}{\ell} & \text{(A-1a)} \\ f_{2_{mn}}(x)\, g_{2_n}(y) = B_{mn} \cos \delta_{mn} \frac{x}{b} \cos \alpha_n \frac{y}{\ell} & \text{(A-1b)} \end{cases}$$

Therefore, in order to perform the indicated Fourier analysis we must obtain the orthogonal functions $\phi_{mn}(x,y)$ associated with the composite flat-plate system. Let us assume that the set of functions $\psi_{mn}(x,y)$ are orthogonal with respect to some "weighting" function and let us assume further that function to be a constant:

$$\phi_{mn}(x,y) = \begin{cases} \phi_{1_{mn}} = C_1\, f_{1_{mn}}(x)\, g_{1_m}(y) & \text{(A-2a)} \\ \phi_{2_{mn}} = C_2\, f_{2_{mn}}(x)\, g_{2_n}(y) & \text{(A-2b)} \end{cases}$$

To prove the orthogonality of the ϕ_{mn}'s:

$$\iint_S \phi_{mn}\, \phi_{pq}\, dS = \int_0^{\ell}\!\!\int_0^{a} \phi_{1_{mn}}\, \phi_{1_{pq}}\, dx\, dy + \int_0^{\ell}\!\!\int_0^{b} \phi_{2_{mn}}\, \phi_{2_{pq}}\, dx\, dy = 0 \qquad \text{(A-3)}$$

if $mn \neq pq$

Since the ψ_{mn}'s satisfy equations (26), so must the ϕ_{mn}'s and ϕ_{pq}'s , therefore:

$$\kappa \frac{\partial^2 \phi_{mn}}{\partial x^2} + \kappa \frac{\partial^2 \phi_{mn}}{\partial y^2} + \lambda_{mn}^2 \phi_{mn} = 0 \qquad \text{(A-4a)}$$

$$\varkappa \frac{\partial^2 \phi_{pq}}{\partial x^2} + \varkappa \frac{\partial^2 \phi_{pq}}{\partial y^2} + \lambda^2_{pq} \phi_{pq} = 0 \qquad \text{(A-4b)}$$

Now let us multiply equation (A-4a) by ϕ_{pq} and equation (A-4b) by ϕ_{mn} and subtract to obtain:

$$(\lambda^2_{mn} - \lambda^2_{pq})(\phi_{mn}\phi_{pq}) = \varkappa\phi_{mn}\left(\frac{\partial^2 \phi_{pq}}{\partial x^2} + \frac{\partial^2 \phi_{pq}}{\partial y^2}\right) - \varkappa\phi_{pq}\left(\frac{\partial^2 \phi_{mn}}{\partial x^2} + \frac{\partial^2 \phi_{mn}}{\partial y^2}\right) \qquad \text{(A-5)}$$

Integrating equation (A-5) we have, after expanding the right hand side:

$$(\lambda^2_{mn} - \lambda^2_{pq})\iint_S \phi_{mn}\phi_{pq}\, dS = \int_0^l\int_0^a \varkappa_1\left\{\phi_{1mn}\left(\frac{\partial^2 \phi_{1pq}}{\partial x^2} + \frac{\partial^2 \phi_{1pq}}{\partial y^2}\right) - \phi_{1pq}\left(\frac{\partial^2 \phi_{1mn}}{\partial x^2} + \frac{\partial^2 \phi_{1mn}}{\partial y^2}\right)\right\} dx\, dy + \int_0^l\int_0^b \varkappa_2\left\{\phi_{2mn}\left(\frac{\partial^2 \phi_{2pq}}{\partial x^2} + \frac{\partial^2 \phi_{2pq}}{\partial y^2}\right) - \phi_{2pq}\left(\frac{\partial^2 \phi_{2mn}}{\partial x^2} + \frac{\partial^2 \phi_{2mn}}{\partial y^2}\right)\right\} dx\, dy \qquad \text{(A-6)}$$

Considering just the first integration on the right hand side of equation (A-6) we substitute from equations (A-2) to obtain:

$$\int_0^l\int_0^a (\varkappa_1 C_1^2)\left\{f_{1mn}g_{1m}\left[g_{1p}\frac{d^2 f_{1pq}}{dx^2} - f_{1pq}\frac{d^2 g_{1p}}{dx^2}\right] - f_{1pq}g_{1p}\left[g_{1m}\frac{d^2 f_{1mn}}{dx^2} + f_{1mn}\frac{d^2 g_{1m}}{dy^2}\right]\right\} dx\, dy \qquad \text{(A-7)}$$

partial integration yields:

$$(\varkappa_1 C_1^2)\left[f_{1mn}\frac{df_{1pq}}{dx} - f_{1pq}\frac{df_{1mn}}{dx}\right]_{x=0}^{x=a}\int_0^l g_{1m}g_{1p}\, dy +$$

$$(\varkappa_1 C_1^2)\left[g_{1m}\frac{dg_{1p}}{dy} - g_{1p}\frac{dg_{1m}}{dy}\right]_{y=0}^{y=l}\int_0^a f_{1mn}f_{1pq}\, dx \qquad \text{(A-8)}$$

Recalling our boundary conditions:

$$\left.\frac{df_{1mn}}{dx}\right|_{x=0} = \left.\frac{df_{1pq}}{dx}\right|_{x=0} = 0 \qquad \text{(A-9)}$$

$$\left.\frac{dg_{1m}}{dy}\right|_{y=0} = \left.\frac{dg_{1p}}{dy}\right|_{y=0} = 0 \tag{A-10}$$

$$\left.\frac{dg_{1m}}{dy}\right|_{y=\ell} = \frac{h}{K_1}\, g_{1m}(\ell) \tag{A-11a}$$

$$\left.\frac{dg_{1p}}{dy}\right|_{y=\ell} = \frac{h}{K_1}\, g_{1p}(\ell) \tag{A-11b}$$

Therefore, the only contribution to the integral is at x = a:

$$\varkappa_1 C_1^2 \int_0^\ell g_{1m}\, g_{1p} \left\{ f_{1mn}(a) \left.\frac{df_{1pq}}{dx}\right|_{x=a} - f_{2pq}(a) \left.\frac{df_{1mn}}{dx}\right|_{x=a} \right\} dy \tag{A-12a}$$

Likewise, from the second integral on the right hand side of (A-6) we may obtain:

$$\varkappa_2 C_2^2 \int_0^\ell g_{2n}\, g_{2q} \left\{ f_{2mn}(b) \left.\frac{df_{2pq}}{dx}\right|_{x=b} - f_{2pq}(b) \left.\frac{df_{2mn}}{dx}\right|_{x=b} \right\} dy \tag{A-12b}$$

At the interface:

$$f_{1mn}(a)\, g_{1m} = f_{2mn}(b)\, g_{2n} \quad \text{, and} \tag{A-13}$$

$$K_1 g_{1m} \left.\frac{df_{1mn}}{dx}\right|_{x=a} = -K_2\, g_{2n} \left.\frac{df_{2mn}}{dx}\right|_{x=b} \tag{A-14}$$

Also

$$f_{1pq}(a)\, g_{1p} = f_{2pq}(b)\, g_{2q} \quad \text{and} \tag{A-13a}$$

$$K_1 g_{1p} \left.\frac{df_{1pq}}{dx}\right|_{x=a} = -K_2\, g_{2q} \left.\frac{df_{2pq}}{dx}\right|_{x=b} \tag{A-14a}$$

Now substituting equations (A-13) into (A-12a) we obtain:

$$\varkappa_1 C_1^2 \int_0^\ell -g_{2n}\, g_{2q} \left\{ f_{2m}(b) \frac{K_2}{K_1} \left.\frac{df_{2pq}}{dx}\right|_{x=b} - f_{2pq}(b) \frac{K_2}{K_1} \left.\frac{df_{2mn}}{dx}\right|_{x=b} \right\} dy \tag{A-15}$$

Now adding (A-15) to (A-12b) we obtain:

$$\int_0^\ell g_{2n}\, g_{2q} \left\{ f_{2mn}(b) \left.\frac{df_{2pq}}{dx}\right|_{x=b} - f_{2pq}(b) \left.\frac{df_{2mn}}{dx}\right|_{x=b} \right\} \left(\varkappa_2 C_2^2 - \varkappa_1 C_1^2 \frac{K_2}{K_1} \right) dy \tag{A-16}$$

Since we would like this integral to be identically zero everywhere, let us assume:

$$\varkappa_2 C_2^2 - \varkappa_1 C_1^2 \frac{K_2}{K_1} = 0 \qquad \text{(A-17)}$$

or

$$\frac{C_1^2}{C_2^2} = \frac{\varkappa_2}{\varkappa_1} \frac{K_1}{K_2}$$

We may conclude that:

$$C_1 = \sqrt{\frac{K_1}{\varkappa_1}} = \sqrt{\rho_1 c_1} \qquad \text{(A-18a)}$$

and

$$C_2 = \sqrt{\frac{K_2}{\varkappa_2}} = \sqrt{\rho_2 c_2} \qquad \text{(A-18b)}$$

Which are the same functions obtained by Mayer (ref. (4)) for the one-dimensional composite slab problem. Substituting the appropriate values for C_1 and C_2 into equation (A-16) we see that this integral is everywhere zero. Therefore, from equation (A-6) we have:

$$(\lambda_{mn}^2 - \lambda_{pq}^2) \iint_S \phi_{mn} \phi_{pq} \, dS = 0 \qquad \text{(A-19)}$$

But

$$\lambda_{mn}^2 \neq \lambda_{pq}^2 \qquad \text{if } mn \neq pq$$

Therefore equation (A-3) has been satisfied and the orthogonality of the functions described by equations (A-2) has been established. In order to determine the coefficients of the expansion ψ_{mn}, we should multiply equation (A-1) by $C\,\phi_{pq}$ and integrate over the region of interest to obtain:

$$\iint_S C\,\psi_{mn} \phi_{pq} \, dS = \iint_S \phi_{mn} \phi_{pq} \, dS = \begin{cases} 0 & mn \neq pq \\ C^2 \iint_S \psi_{mn}^2 \, dS & mn = pq \end{cases}$$

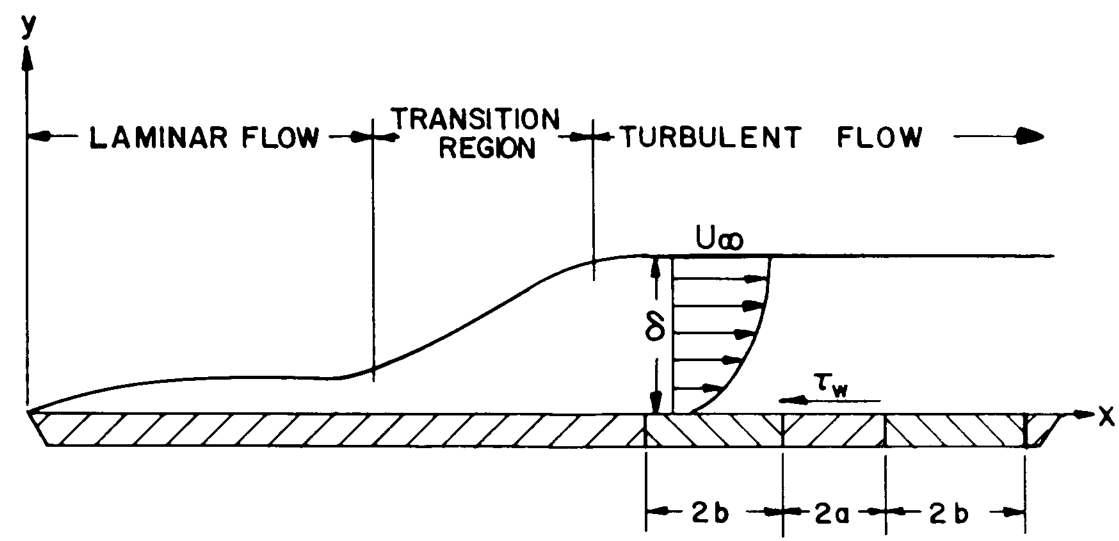


FIGURE 1. BOUNDARY LAYER OVER A COMPOSITE FLAT PLATE.

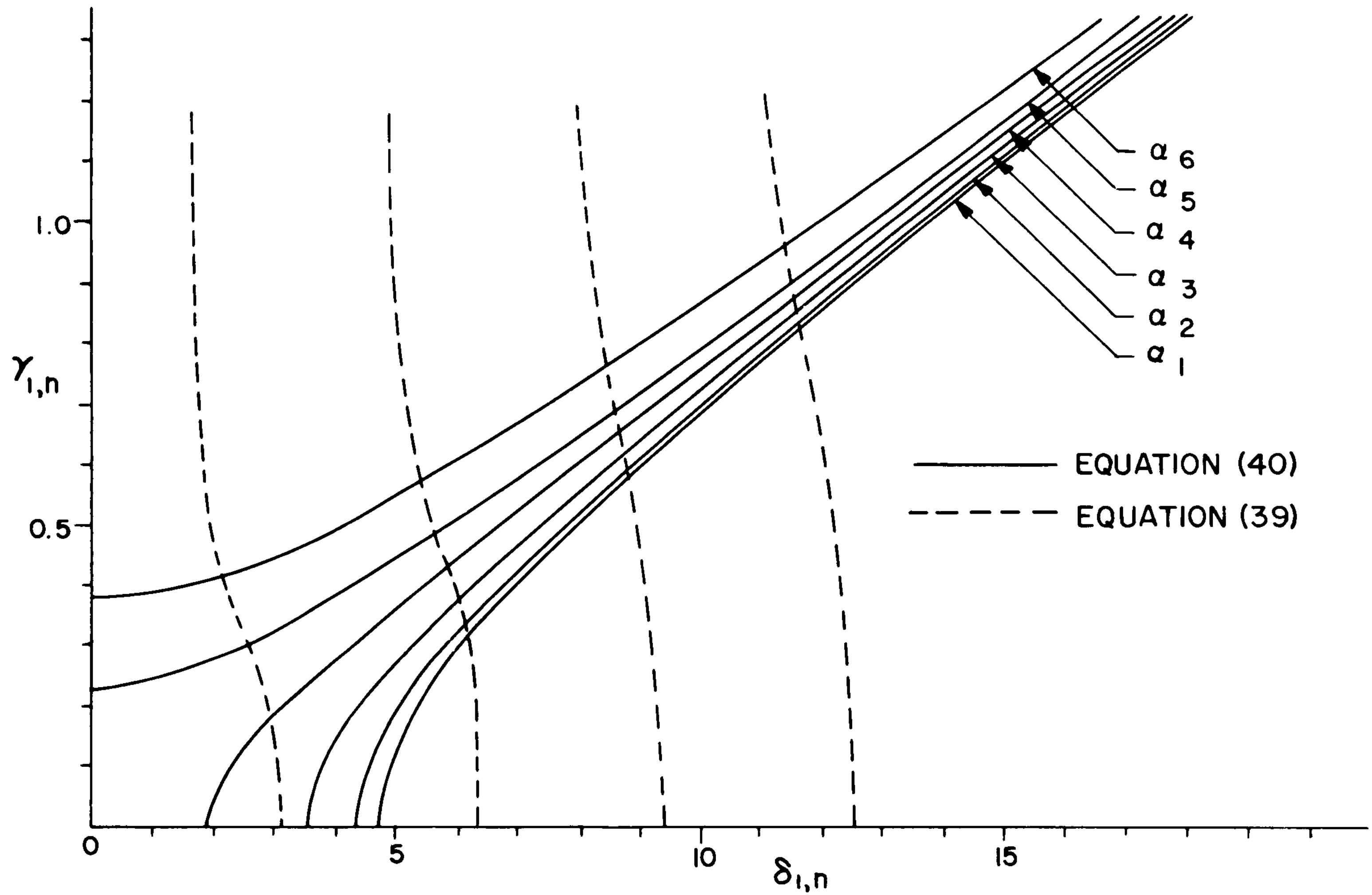


FIGURE 2. DETERMINATION OF EIGENVALUES

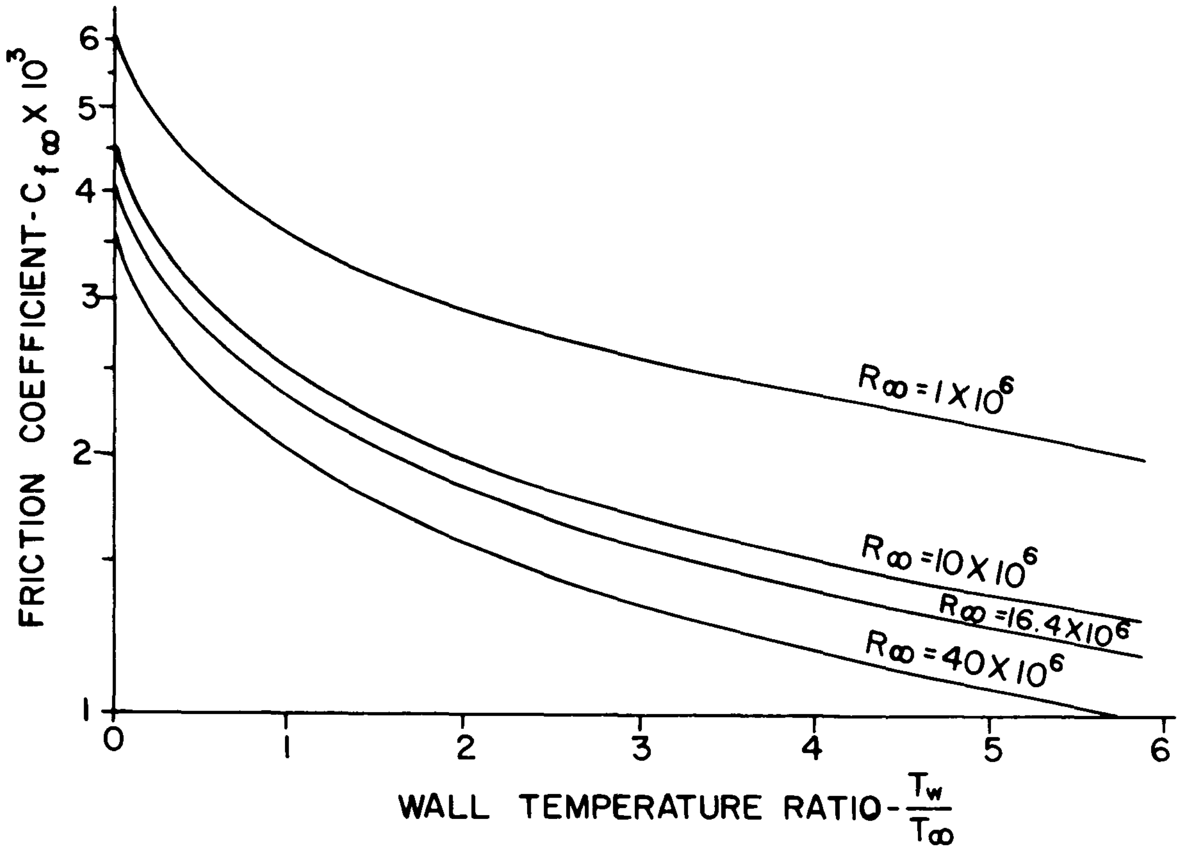


FIGURE 3. LOCAL TURBULENT FRICTION COEFFICIENT AS A FUNCTION OF WALL TEMPERATURE AT MACH I

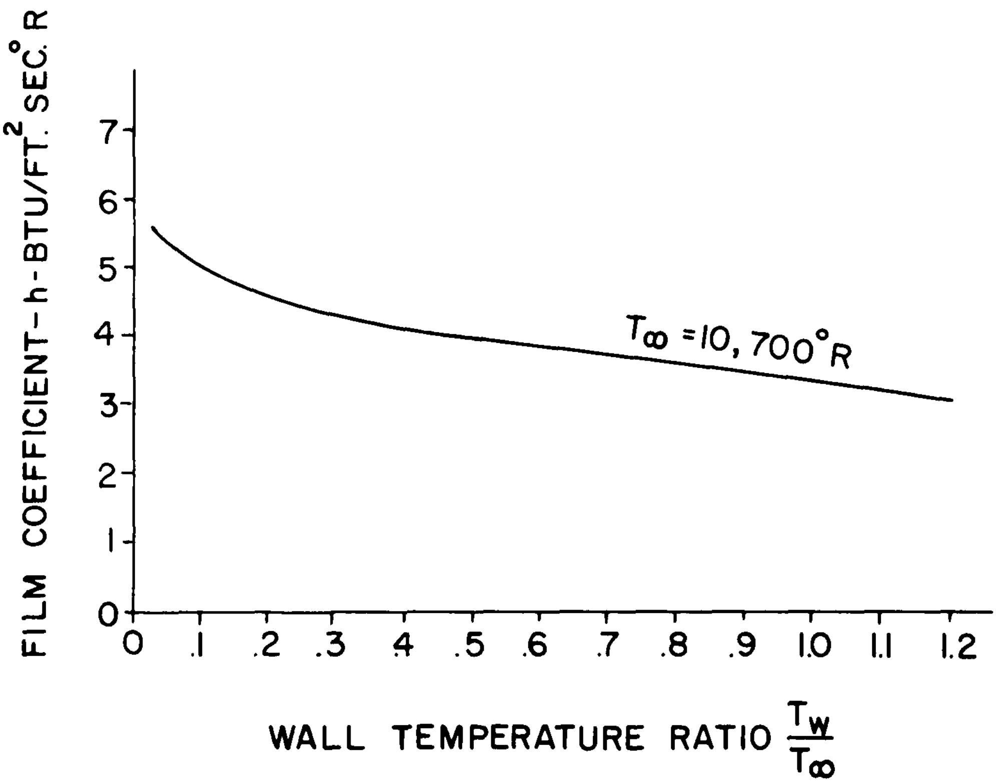


FIGURE 4 LOCAL TURBULENT FILM COEFFICIENT AS A FUNCTION OF WALL TEMPERATURE AT MACH 1

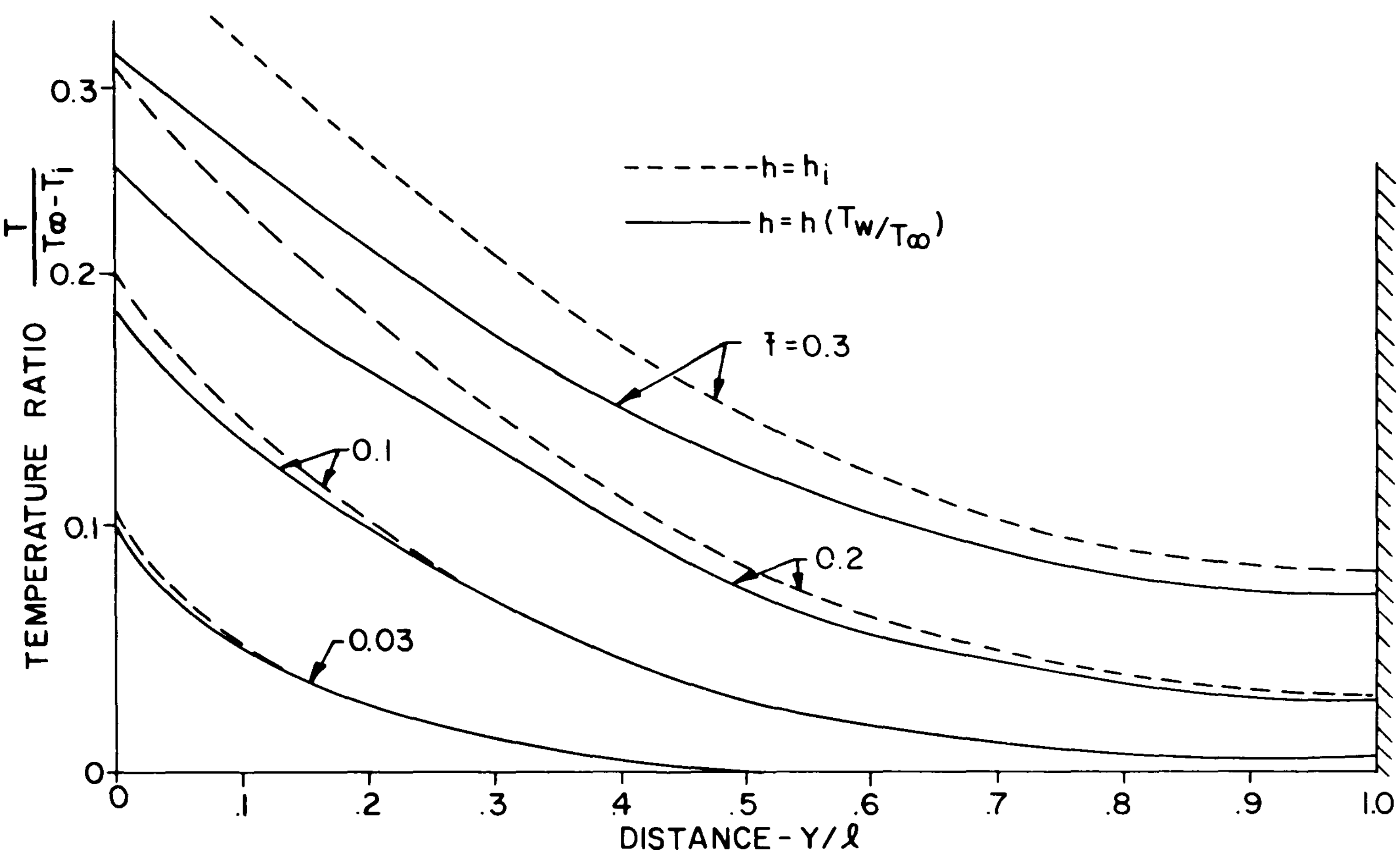


FIGURE 5. TEMPERATURE DISTRIBUTION THROUGH A FLAT PLATE
($M=1$, $T_\infty = 10{,}700°R$, $R_\infty = 16.4 \times 10^6$)

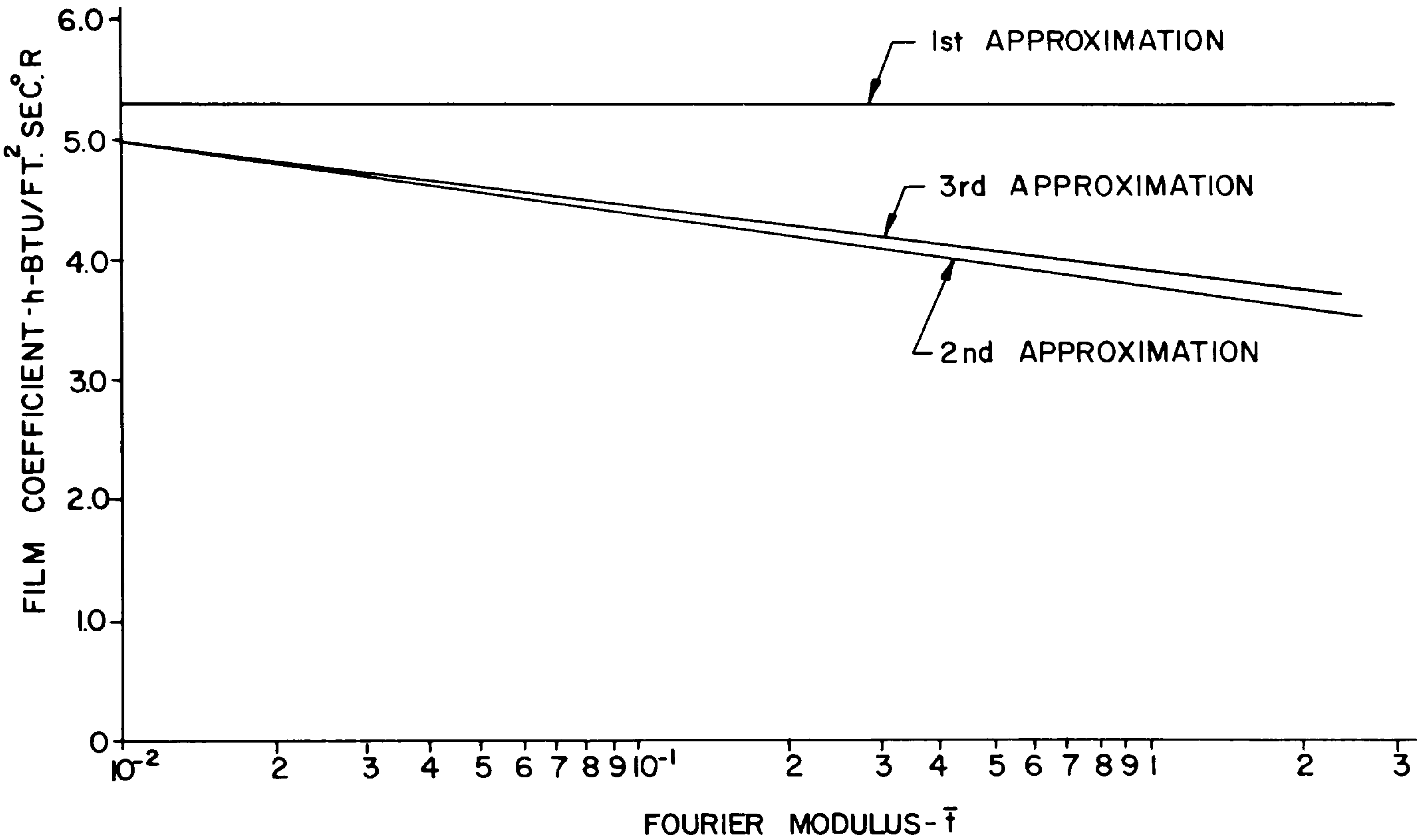


FIGURE 6 TRANSIENT TURBULENT FILM COEFFICIENT
($M = 1$, $T_\infty = 10{,}700°R$, $R_\infty = 16.4 \times 10^6$)

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SUBJECT ANALYSIS OF REPORT

DESCRIPTORS	CODES	DESCRIPTORS	CODES	DESCRIPTORS	CODES
Aerodynamic heating	AERY	Contact	CONA	Temperature	[illegible]
Composite	COMS	Exposure	EXOS	Ratio	RATI
Flat	FLAT	Boundary layer	BOUL	Distribution	[illegible]
Plate	PLAT	Fourier	FOUR	Eigenvalues	[illegible]
Heat	HEAT	Series	SERE	Heat transfer	[illegible]
Conduction	COND	Orthogonal	ORTG		
Equation	EQUA	Mayer	MAYE		
One-dimensional	ONED	Van Driest	VANR		
Two-dimensional	TWOD	Turbulence	TUBU		
Slabs	SLAB	Theory	THEY		
Variation	VART	Errors	ERRO		
Thermal	THER	Surface	SURA		

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Naval Ordnance Laboratory. White Oak, Md.
(NOL technical report 63-163).
THE AERODYNAMIC HEATING OF A COMPOSITE FLAT PLATE, (U) by James L. Rand. 12 July 1963. v.p. charts. (Ballistics research report 113)
UNCLASSIFIED

The purpose of this study was to find solutions to the heat-conduction equation in both one and two dimensions when applied to a composite flat plate. The two-dimensional model is that of two slabs with different thermal properties, in good thermal contact with each other while being exposed to the same boundary layer. The Van Driest turbulent boundary-layer theory has been applied to the one-dimensional problem to determine the film coefficient required by the boundary condition.

Abstract card is unclassified

1. Plates, Flat - Aerodynamic heating
2. Plates, Flat - Boundary layer
3. Heating, Aerodynamic
4. Boundary layer, Turbulent

I. Title
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III. Series

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